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Electromagnetic Scattering from Turbulent Boundary Layers

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ELECTROMAGNETIC SCATTERING FROM TURBULENT BOUNDARY LAYERS

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ABSTRACT

The incoherent scattering cross section of an underdense turbulent boundary layer adjacent to both a perfectly conducting ground plane and a perfectly conducting cylindrical surface is determined. It is found that the effect of the metallic surfaces on scattering from the turbulent plasma depends on the thickness of the boundary layer and the characteristics of the incident electromagnetic waves. For optically thin boundary layers, a plane metallic surface will enhance scattering in one linear polarization by up to 12 dB while eliminating scattering in the other polarization.

Accepted for the Air Force
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ELECTROMAGNETIC SCATTERING FROM TURBULENT BOUNDARY LAYERS

I. INTRODUCTION

Scattering of electromagnetic (EM) waves from plasmas residing in turbulent media has largely been treated as a scalar problem rather than a vector problem. In the vector problem, the radar cross section (RCS) of a turbulent medium is determined from a coherent summation over the rays scattered from the medium, whereas in the scalar problem, the phase of the scattered ray is ignored and an incoherent summation is used. Generally, polarization is also ignored in the scalar problem because the incoherent summation makes all polarizations equivalent.

Scattering of EM waves from wakes of hypersonic vehicles has been treated as a scalar problem because the turbulent fluctuations in the "thick" wake rapidly randomize the phase of any signal propagating through the medium. However, the thickness of turbulent boundary layers on hypersonic vehicles is often much smaller than the wavelength of the incident EM waves and one would not expect phase to be randomized. Therefore, a vector solution is appropriate.

Yakimenko[1] has suggested a very convenient relationship for obtaining such vector solutions. He applied his technique to "thick" turbulent media and discontinuous media. A generalized form of his relationship derived in the Appendix may be used to investigate "thick" as well as "thin" media. In Section II we show that vector solutions are essentially limited to cases where the Born Approximation is valid, and that the Born Approximation always requires a vector rather than a scalar solution.* Then in Sections III and IV the results are applied to a turbulent boundary layer on a ground plane and on a cylinder.

* Note that if the EM scattering process can be fully described by a single ray in the high frequency limit, then vector and scalar solutions are equivalent.

II. SCATTERING FROM TURBULENT MEDIA IN THE GEOMETRICAL OPTICS LIMIT

Limits on the validity of vector solutions and scalar solutions may be established in the WKB limit using Eq. (1) which includes both vector and scalar solutions as special cases. Equation (1) is derived in the Appendix. For our purposes it is sufficient to regard the constitutive parameter $\overleftrightarrow{K}(r)$ as a scalar $\cdot K(\vec{r}) = \epsilon(\vec{r})/\epsilon_o$. Equation (1) is valid for scattering from a weakly turbulent medium ($\Delta \cdot \ln K(\vec{r}) << 1$ and $\partial/\partial t\, K(\vec{r}) << \partial/\partial t\, E(\vec{r})$ in which the electric field $E(\vec{r})$ is completely continuous except at perfectly conducting surfaces.

$$\langle\sigma(\vec{k}_s|\vec{k}_i)\rangle = \frac{k^4}{4\pi}\int d^3\vec{r}\int d^3\vec{r}'\ \langle\delta K(\vec{r})\ \delta K(r')\ \vec{E}(\vec{r})$$

$$\cdot\ \vec{U}(\vec{r})\ \vec{E}(\vec{r}')\cdot\vec{U}(\vec{r}')\rangle \quad , \tag{1}$$

where $\delta K(\vec{r}) = K(\vec{r}) - 1$, and where $\vec{E}(\vec{r})$ is the solution to the propagation problem for a plane wave incident in the $\vec{k}_i$ direction on the configuration of perfect conductors and turbulent plasma media. $\vec{U}(\vec{r})$ is the solution to a special propagation problem of a plane wave incident in the $-\vec{k}_s$ direction on the same configuration of perfect conductors but with the plasma replaced with free-space. The function $\vec{U}(\vec{r})$ is essentially the Green's function of the scattering problem evaluated at $r' \to \infty$.

In the geometrical optics limit, the solutions to the propagation problems may be expressed as a summation of rays,

$$\vec{U}(\vec{r}) = \sum_n \hat{p}_n U_n(\vec{r}) \exp\left\{-i\vec{k}_n\cdot\left(\vec{r} + \int^{\vec{r}} \delta K(\vec{\tau})\, d\vec{\tau}\right)\right\} \quad , \tag{2}$$

$$\vec{E}(\vec{r}) = \sum_m \hat{p}_m E_m(\vec{r}) \exp\left\{i\vec{k}_m\cdot\left(\vec{r} + \int^{\vec{r}} \delta K(\vec{\tau})\, d\vec{\tau}\right)\right\} \quad , \tag{3}$$

where $\hat{p}_m$ and $\hat{p}_n$ are the polarization vectors of each ray and the path integrals are unique for each ray. Upon substitution of Eqs. (2) and (3) into Eq. (1), one obtains

$$\langle\sigma(\vec{k}_s|\vec{k}_i)\rangle = \frac{k^4}{4\pi} \sum_{m,m',n,n'} (\hat{p}_m \cdot \hat{p}_n)(\hat{p}_{m'} \cdot \hat{p}_{n'})$$

$$\times \int d^3\vec{r} \Big\langle E_m(\vec{r})\, E^*_{m'}(\vec{r})\, U_n(\vec{r})\, U^*_{n'}(\vec{r})$$

$$\times \exp\left\{ i(\vec{k}_m - \vec{k}_{m'} - \vec{k}_n + \vec{k}_{n'}) \cdot \left(\vec{r} + \int^{\vec{r}} \delta K(\vec{\tau})\, d\vec{\tau}\right)\right\}$$

$$\times \int d^3\Delta\vec{r}\, \delta K(\vec{r})\, \delta K(\vec{r}+\Delta\vec{r}) \exp\Bigg\{ i(\vec{k}_m - \vec{k}_n)$$

$$\cdot \left(\Delta\vec{r} + \int^{\vec{r}+\Delta\vec{r}} \delta K(\vec{\tau})\, d\vec{\tau} - \int^{\vec{r}} \delta K(\vec{\tau})\, d\vec{\tau}\right)\Bigg\}\Big\rangle , \tag{4}$$

where $\Delta\vec{r} = \vec{r} - \vec{r}'$, and $E_m(\vec{r}+\Delta\vec{r}) \doteq E_m(\vec{r})$ and $U_n(\vec{r}+\Delta\vec{r}) \doteq U_n(\vec{r})$ is assumed. In order to relate Eq. (4) to the conventional spectrum function:

$$S(\vec{k}) \equiv \int d^3\Delta r\, \langle \delta K(\vec{r})\, \delta K(\vec{r}+\Delta\vec{r})\rangle \exp\{i\vec{k}\cdot\Delta\vec{r}\} \quad , \tag{5}$$

it is necessary that

$$\exp\left\{ i(\vec{k}_m - \vec{k}_n) \cdot \left(\int^{\vec{r}+\Delta\vec{r}} \delta K(\vec{\tau})\, d\vec{\tau} - \int^{\vec{r}} \delta K(\vec{\tau})\, d\vec{\tau}\right)\right\} << 1 \quad ,$$

or conservatively,

$$\langle \delta K(\vec{r})^2 \rangle\, k\Lambda << 2 \quad , \tag{6}$$

where Λ is the turbulence scale size and $|\vec{k}_m - \vec{k}_n| \leqslant k$. Equation (6) is consistent with the treatment of turbulence as weak fluctuations in a laminar medium in the Appendix.

The two limiting cases of a vector solution and a scalar solution may be established by further examination of Eq. (4). A vector solution is obtained if the lengths of all ray paths in the medium are so short that phase is not randomized by the weak fluctuations. This limit is given by

$$\exp\left\{i(\vec{k}_m - \vec{k}_{m'} - \vec{k}_n + \vec{k}_{n'}) \cdot \int^{\vec{r}} \delta K(\vec{\tau})\, d\vec{\tau}\right\} << 1 \quad ,$$

or conservatively,

$$\langle \delta K(\vec{r})^2 \rangle\, k\ell << 1 \quad , \tag{7}$$

where ℓ is the longest ray path in the plasma medium. It is interesting to compare this to the conditions for validity of the Born Approximation. Salpeter and Treiman[2] indicate that the restriction on use of the Born Approximation is usually the requirement that the incident beam is not diffused over a wide angle $\langle \delta\Theta^2 \rangle$ by small angle scatterings. Since $\langle \delta\Theta^2 \rangle = \langle \delta K(r)^2 \rangle\, \ell/\Lambda$, the Born Approximation is valid in the region

$$\langle \delta K(\vec{r})^2 \rangle\, \ell << \Lambda \quad . \tag{8}$$

Because scale size Λ is always small compared to ℓ, Eq. (8) is more restrictive than Eq. (7) and the Born Approximation always requires a vector solution, but the converse is not necessarily true. In the limit given by Eq. (7), Eq. (1) reduces to Yakimenko's relationship,

$$\langle \sigma(\vec{k}_s | \vec{k}_i) \rangle = \frac{k^4}{4\pi} \int d^3\vec{r} \int d^3\vec{r}'\, \langle \delta K(\vec{r})\, \delta K(\vec{r}') \rangle\, \vec{U}(\vec{r}) \cdot \vec{E}(\vec{r})\, \vec{U}^*(\vec{r}') \cdot \vec{E}^*(\vec{r}') \quad . \tag{9}$$

A scalar solution is obtained from Eq. (1) in the opposite limit that ray paths are long and phase is randomized. In this case,

$$\exp\left\{i(\vec{k}_m - \vec{k}_{m'} - \vec{k}_n + \vec{k}_{n'}) \cdot \int^{\vec{r}} \delta K(\vec{\tau})\, d\vec{\tau}\right\} >> 1 \tag{10}$$

for all values of m, m', n, and n' except m = m' and n = n'. This limit is obtained if $\langle \delta K(\vec{r})^2 \rangle k\ell >> 1$ for all ray paths where Eq. (1) reduces to

$$\langle \sigma(\vec{k}_s | \vec{k}_i) \rangle = \frac{k^4}{4\pi} \langle \delta K(\vec{r})^2 \rangle \sum_m \sum_n (\hat{p}_m \cdot \hat{p}_n)^2 \, S(\vec{k}_m - \vec{k}_n)$$

$$\times \int d^3\vec{r}\, I_m(\vec{r})\, I_n(\vec{r}) \quad , \tag{11}$$

where $I_m(\vec{r}) = E_m(\vec{r})\, E^*_m(\vec{r})$, and $I_n(\vec{r}) = U_n(\vec{r})\, U^*_n(\vec{r})$. The scalar solution is essentially the vector solution with cross terms removed. The ray intensities have been left as functions of position because diffusion of the incident beam may be important when $\langle \delta K(r)^2 \rangle k\ell >> 1$ in a weakly turbulent medium.

III. SCATTERING FROM TURBULENT BOUNDARY LAYERS ON GROUND PLANES

The thicknesses of turbulent boundary layers are normally sufficiently small for a vector solution to be valid, except perhaps near grazing angles of incidence. Although the condition for the validity of the Born Approximation, $\langle \delta K(r)^2 \rangle \ell << \Lambda$, is even more restrictive than Eq. (7), we choose to use the Born Approximation for mathematical convenience. In the Born limit, the integrand of Eq. (9) is given by Eqs. (2) and (3) with $\delta K(\vec{\tau}) \doteq 0$ and $U_n(\vec{r}) = E_m(\vec{r}) = 1$.

<u>Horizontal Polarization</u> (electric vector normal to the plane of incidence)

$$\vec{E}(\vec{r}) \cdot \vec{U}(\vec{r}) = \left[e^{i\vec{k}_i \cdot \vec{r}} - e^{i\vec{k}'_i \cdot \vec{r}}\right] \left[e^{-i\vec{k}_s \cdot \vec{r}} - e^{-i\vec{k}'_s \cdot \vec{r}}\right] \quad . \tag{13}$$

Vertical Polarization

$$\vec{E}(\vec{r}) \cdot \vec{U}(\vec{r}) = \cos\Theta_s \cos\Theta_i \left[e^{i\vec{k}_i \cdot \vec{r}} - e^{i\vec{k}_i' \cdot \vec{r}}\right]\left[e^{-i\vec{k}_s \cdot \vec{r}} - e^{-i\vec{k}_s' \cdot \vec{r}}\right]$$

$$+ \sin\Theta_s \sin\Theta_i \left[e^{i\vec{k}_i \cdot \vec{r}} + e^{i\vec{k}_i' \cdot \vec{r}}\right]$$

$$\times \left[e^{-i\vec{k}_s \cdot \vec{r}} + e^{-i\vec{k}_s' \cdot \vec{r}}\right] \quad , \tag{14}$$

where

$$\vec{k}_i = k(\hat{x} \sin\Theta_i - \hat{y} \cos\Theta_i) \quad ,$$

$$\vec{k}_i' = k(\hat{x} \sin\Theta_i + \hat{y} \cos\Theta_i) \quad ,$$

$$\vec{k}_s = -k(\hat{x} \sin\Theta_s - \hat{y} \cos\Theta_s) \quad ,$$

$$\vec{k}_s' = -k(\hat{x} \sin\Theta_s + \hat{y} \cos\Theta_s) \quad ,$$

and Θ_i and Θ_s are measured from the normal to the ground plane $\hat{y}$.

Upon substitution into Eq. (10) one obtains

Horizontal Polarization

$$\sigma_H = \left(\frac{k^4}{4\pi}\right) \langle \delta K^2 \rangle \, \mathrm{Ad} \left\{C_1^+ S^- + C_1^- S^+\right\} \quad . \tag{15}$$

Vertical Polarization

$$\sigma_V = \left(\frac{k^4}{4\pi}\right) \langle \delta K^2 \rangle \, \mathrm{Ad} \Big[\cos^2\Theta_i \cos^2\Theta_s \left\{C_1^+ S^- + C_1^- S^+\right\}$$

$$+ \sin^2\Theta_i \sin^2\Theta_s \left\{C_2^+ S^- + C_2^- S^+\right\}$$

$$+ \sin\Theta_i \sin\Theta_s \cos\Theta_i \cos\Theta_s \left\{C_3^+ S^- + C_3^- S^+\right\}\Big] \quad , \tag{16}$$

where

$$S^{\pm} = S\left(2k \cos \frac{\Theta_i \pm \Theta_s}{2}\right)$$

$$C_1^{\pm} = 2 + \frac{\sin[2kd(\cos\Theta_i \pm \cos\Theta_s)]}{kd(\cos\Theta_i \pm \cos\Theta_s)} - \frac{\sin[2kd \cos\Theta_i]}{kd \cos\Theta_i}$$

$$- \frac{\sin[2kd \cos\Theta_s]}{kd \cos\Theta_s}$$

$$C_2^{\pm} = 2 + \frac{\sin[2kd(\cos\Theta_i \pm \cos\Theta_s)]}{kd(\cos\Theta_i \pm \cos\Theta_s)} + \frac{\sin[2kd \cos\Theta_i]}{kd \cos\Theta_i}$$

$$+ \frac{\sin[2kd \cos\Theta_s]}{kd \cos\Theta_s}$$

$$C_3^{\pm} = 2 - \frac{\sin[2kd(\cos\Theta_i \pm \cos\Theta_s)]}{kd(\cos\Theta_i \pm \cos\Theta_s)}$$

and d is the thickness of the boundary layer and A is the illuminated surface area.

The properties of this solution become apparent when evaluated in the limit of an optically thin boundary layer $(kd)^2 << 1$ and compared to the cross section σ_o of a boundary layer that is not backed by a ground plane.

$$\sigma_o = \left(\frac{k^4}{4\pi}\right) \langle \delta K^2 \rangle \, A d \, S\left(2k \cos \frac{\Theta_i - \Theta_s}{2}\right) \quad . \tag{17}$$

The spectrum of the turbulence is isotropic because the scale length of the turbulence is small compared to thickness of the boundary layer (i.e., $k\Lambda << 1$). For the example of a Kolmogoroff spectrum,

$$S\left(2k \cos \frac{\Theta_i \pm \Theta_s}{2}\right) = 15.5\,\Lambda^3 \left[1 + 4k^2\Lambda^2 \cos^2\left(\frac{\Theta_i \pm \Theta_s}{2}\right)\right]^{-11/3}$$

$$\underset{k\Lambda << 1}{\sim} 15.5\,\Lambda^3 \quad . \tag{18}$$

For $(kd)^2 \ll 1$ and using Eqs. (17) and (18), Eqs. (15) and (16) reduce to

$$\frac{\sigma_V}{\sigma_o} \underset{(kd)^2 \ll 1}{\sim} 16 \sin^2\Theta_i \sin^2\Theta_s + \frac{16}{3} (kd)^2 \sin\Theta_i \sin\Theta_s \cos^2\Theta_i \cos^2\Theta_s \quad , \tag{19}$$

$$\frac{\sigma_H}{\sigma_o} \underset{(kd)^2 \ll 1}{\sim} \frac{16}{3} (kd)^4 \cos^2\Theta_i \cos^2\Theta_s \quad . \tag{20}$$

We see that the effect of the ground plane is virtually to eliminate horizontally polarized scattering from thin boundary layers while enhancing vertically polarized scattering for angles of incidence and reflection greater than $\pi/4$. If the medium is so underdense that the $\langle \delta K(r)^2 \rangle \, \ell \ll \Lambda$ condition is satisfied for both $\ell = d/\cos\Theta_i$ and $\ell = d/\cos\Theta_s$, then vertical polarization is enhanced by 12 dB at grazing angles of incidence. If circularly polarized waves were incident at grazing angles of incidence, equal principal and orthogonal returns would be scattered with a 9-dB enhancement.

IV. SCATTERING FROM TURBULENT BOUNDARY LAYERS ON PERFECTLY CONDUCTING CYLINDERS

The incoherent scattering from a thin boundary layer wrapped around a large perfectly conducting cylinder shall be determined in the limit that $ka \gg 1$ and $kd \ll 1$, where a is the radius of the cylinder. In this limit, the surface of the cylinder may be assumed to be locally plane and the results of Section III applied. For mathematical convenience only backscattering is considered. At every point on the cylinder the scattering properties of the optically thin boundary layer may be represented as a spectrum function as follows:

$$S_{VV} = 16 \cos^2\alpha_V \sin^4\beta \, S(2k) \quad , \tag{21}$$

$$S_{HH} = 16 \cos^2 \alpha_H \sin^4 \beta \, S(2k) \quad , \tag{22}$$

$$S_{VH} = S_{HV} = 16 \cos \alpha_H \cos \alpha_V \sin^4 \beta (\cos \alpha_H \cos \alpha_V$$

$$+ \sin \alpha_H \sin \alpha_V) \, S(2k) \quad , \tag{23}$$

where

$$\cos \alpha_V = \cos \varphi \, \sin \Theta \quad , \tag{24}$$

$$\cos \alpha_H = \sin \varphi \, \sin \Theta \quad , \tag{25}$$

$$\cos \beta = \cos \varphi \, \cos \Theta \quad , \tag{26}$$

and Θ is the angle between the incident ray and the normal to the cylinder axis (the complement of the aspect angle) and φ is the angular location on the cylinder.

The incoherent cross section is then obtained by integrating φ over the upper surface of the cylinder, the lower half-surface being shadowed.

$$\frac{\sigma_{HH}}{\sigma_o} = 8 \sin^2 \Theta - 6 \cos^2 \Theta \sin^2 \Theta$$

$$- 5 \sin^4 \Theta \cos^2 \Theta (1 - \frac{7}{8} \cos^2 \Theta) \quad , \tag{27}$$

$$\frac{\sigma_{VV}}{\sigma_o} = 8 \sin^2 \Theta + 6 \cos^2 \Theta (1 + \sin^2 \Theta)$$

$$+ \sin^4 \Theta \cos^2 \Theta (1 + \frac{35}{8} \cos^2 \Theta) \quad , \tag{28}$$

$$\frac{\sigma_{VH}}{\sigma_o} = \frac{\sigma_{HV}}{\sigma_o} = 2 \sin^6 \Theta + \frac{1}{2} \sin^4 \Theta \cos^4 \Theta \quad , \tag{29}$$

$$\sigma_o = \left(\frac{k^2}{4\pi} \right) \langle \delta K^2 \rangle \, \pi d b \, S(2k) \quad , \tag{30}$$

where b = length of the cylinder illuminated.

In the cylinder case, $\sigma_{VV} \sim \sigma_{HH}$ at near grazing angles of incidence with 9 dB of enhancement due to the presence of the perfectly conducting cylinder surface. The VV return comes primarily from the top of the cylinder while the HH return comes primarily from the sides of the cylinder. Contrary to the case of a flat ground plane, the turbulent boundary layer is polarizing at broadside incidence.

Backscattering from a turbulent boundary layer on a conical surface for axisymmetric incidence is similar to the cylinder case at grazing angles of incidence except for the lack of a shadow boundary. The point-by-point spectrum functions are given by Eqs. (21), (22), and (23) but with Eqs. (24), (25), and (26) replaced by

$$\cos \alpha_V = \cos \varphi \cos \Theta \cos \gamma - \sin \Theta \sin \gamma \quad , \tag{31}$$

$$\cos \alpha_H = \sin \varphi \cos \Theta \cos \gamma - \sin \Theta \sin \gamma \quad , \tag{32}$$

$$\cos \beta = \cos \varphi \sin \Theta \cos \gamma + \cos \Theta \sin \gamma \quad , \tag{33}$$

where γ is the one-half the vertex angle of the conical surface. After integration over the illuminated surface of the cone, the incoherent cross section is obtained

$$\frac{\sigma_{HH}}{\sigma_o} = \frac{\sigma_{VV}}{\sigma_o} = 8 \cos^4 \gamma - 6 \cos^6 \gamma \sin^2 \gamma \quad , \tag{34}$$

$$\sigma_{VH} = \sigma_{HV} = 0 \quad . \tag{35}$$

Thus at grazing angles of incidence, $\Theta \approx (\pi/2) - \gamma$, the cylinder solution to zero-order in Θ equals the conical solution.

APPENDIX

Consider a plane wave $\vec{E}^i(\vec{r})$ incident on a weakly turbulent medium containing plasma. The medium may be bounded by and/or contain perfectly conducting surfaces. The fields in the turbulent medium must satisfy a wave equation having a variable or fluctuating constitutive parameter $\overleftrightarrow{K}(r, t)$. Following Kodis,[3] an integral equation may be set up for the fields $\vec{E}^s(\vec{r})$ scattered from the medium assuming weak fluctuations:

$$\nabla \cdot \ln \left| \overleftrightarrow{K}(\vec{r}, t) \right| \ll 1 \quad , \quad \frac{\partial}{\partial t} \left| \overleftrightarrow{K}(\vec{r}, t) \right| \ll \frac{\partial}{\partial t} \left| \vec{E}(\vec{r}, t) \right| \quad ,$$

$$\vec{E}^s(\vec{r}) = \int d^3\vec{r}\, \overleftrightarrow{\Gamma}(\vec{r} \mid \vec{r}') \cdot \vec{J}(\vec{r}')$$

$$+ \text{(integrals over surfaces of discontinuous } \vec{E}(\vec{r})) \qquad \text{(A-1)}$$

where $\nabla \times \nabla \times \overleftrightarrow{\Gamma}(\vec{r} \mid \vec{r}') - k^2 \overleftrightarrow{\Gamma}(\vec{r} \mid \vec{r}') = \overleftrightarrow{I}\, \delta(\vec{r} - \vec{r}')$ with $\hat{n} \times \overleftrightarrow{\Gamma} = 0$ at the conducting surfaces defines the dyadic Green's function, where

$$\vec{J} = k^2 (\overleftrightarrow{K}(\vec{r}) - \overleftrightarrow{I}) \cdot \vec{E}(\vec{r}) \quad , \qquad \text{(A-2)}$$

and where $\vec{E}(\vec{r})$ is the total field in the turbulent medium.

The treatment of the surface integral in (A-1) is not understood. Therefore, the following derivation is restricted to problems where sharp interfaces between the plasma and vacuum do not occur. The dyadic Green's function may be determined by writing Eq. A-1 in a form that is equivalent to the reciprocity theorem.

$$\hat{p}_s \cdot \int d^3\vec{r} \left\{ \vec{E}^s(\vec{r}')\, \delta(\vec{r} - \vec{r}') - \overleftrightarrow{\Gamma}(\vec{r} \mid \vec{r}') \cdot \vec{J}(\vec{r}') \right\} \quad . \qquad \text{(A-3)}$$

Since $\vec{E}^s(\vec{r})$ may be considered to be the field produced by the source $\vec{J}(\vec{r})$ in the original configuration of plasma media and perfectly conducting surfaces

but with the plasma medium replaced by free space, then

$$\vec{E}_o(\vec{r}\,|\,\vec{r}\,') = \hat{p}_s \cdot \overleftrightarrow{\Gamma}(\vec{r}\,|\,\vec{r}\,') \tag{A-4}$$

must be the field produced by a source $\hat{p}_s \delta(\vec{r}\,' - \vec{r})$ in the same configuration of conducting surfaces and free-space. Here $\hat{p}_s$ is the polarization vector of the scattered fields. Barring discontinuous fields, Eq. (A-1) may then be written as

$$\hat{p}_s \cdot \vec{E}^s(\vec{r}) = \int d^3\vec{r}\,' \, \vec{E}_o(\vec{r}\,'\,|\,\vec{r}) \cdot \vec{J}(\vec{r}\,') \quad , \tag{A-5}$$

and the incoherent scattering cross section of the turbulent medium may immediately be obtained from Eq. (A-5)

$$\langle \sigma(\vec{k}_s\,|\,\vec{k}_i) \rangle = 4\pi |\vec{r}\,|^2 \left\langle \left| \frac{\hat{p}_s \cdot \vec{E}^s(\vec{r})}{\hat{p}_i \cdot \vec{E}^i(\vec{r})} \right|^2 \right\rangle \quad ,$$

$$\langle \sigma(\vec{k}_s\,|\,\vec{k}_i) \rangle = \frac{4\pi |\vec{r}\,|^2}{|\hat{p}_i \cdot \vec{E}^i(\vec{r})|^2} \int d^3\vec{r}\,' \int d^3\vec{r}\,''$$

$$\langle \vec{E}_o(\vec{r}\,|\,\vec{r}\,') \cdot \vec{J}(\vec{r}) \, \vec{E}_o(\vec{r}\,|\,\vec{r}\,'') \cdot \vec{J}(\vec{r}\,'') \rangle \quad . \tag{A-6}$$

It is mathematically convenient to approximate $\vec{E}_o(\vec{r}\,|\,\vec{r}\,')$ in the neighborhood of the plasma medium by the solution $\vec{U}(\vec{r})$ to the propagation problem for a plane wave incident in the direction $-\vec{k}_s$ (from the direction of the source at $\vec{r}$ as $\vec{r} \to \infty$) upon the free-space configuration.

$$\vec{E}_o(\vec{r}\,|\,\vec{r}\,') \underset{\vec{r} \to \infty}{\sim} \frac{\vec{U}(\vec{r}\,')}{4\pi |\vec{r}\,|} \, e^{i\vec{k}_s \cdot \vec{r}} \quad . \tag{A-7}$$

Then using Eq. A-2 and assuming all incident fields have unit magnitude, a generalization of Yakimenko's relationship is obtained.

$$\langle\sigma(\vec{k}_s \mid \vec{k}_i)\rangle = \left(\frac{k^2}{4\pi}\right) \int d^3\vec{r}'' \int d^3\vec{r}' \ \langle[\delta \overleftrightarrow{K}(\vec{r}'') \cdot \vec{E}(\vec{r}'') \cdot \vec{U}(\vec{r}'')]$$

$$\times [\delta \overleftrightarrow{K}^*(\vec{r}') \cdot \vec{E}^*(\vec{r}') \cdot \vec{U}^*(\vec{r}')]\rangle \quad , \qquad \text{(A-8)}$$

where $\delta \overleftrightarrow{K}(\vec{r}') = \overleftrightarrow{K}(\vec{r}') - I$ and $\vec{E}(\vec{r}')$ is the solution to the propagation problem in the configuration of plasma medium plus perfectly conducting surfaces for incidence in the $\vec{k}_i$ direction, and $\vec{U}(\vec{r}')$ is the solution to the propagation problem in the same configuration but with incidence in the $-\vec{k}_s$ direction and the plasma medium replaced by free-space.

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13. ABSTRACT

The incoherent scattering cross section of an underdense turbulent boundary layer adjacent to both a perfectly conducting ground plane and a perfectly conducting cylindrical surface is determined. It is found that the effect of the metallic surfaces on scattering from the turbulent plasma depends on the thickness of the boundary layer and the characteristics of the incident electromagnetic waves. For optically thin boundary layers, a plane metallic surface will enhance scattering in one linear polarization by up to 12 dB while eliminating scattering in the other polarization.

14. KEY WORDS

electromagnetic scattering
underdense turbulent boundary layers